Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Course: Mathematics 1 – A Date: January 4 , 2020 Duration: **3** hours

The Exam consists of two pages Answer **All** Questions No. of questions: 4 Total Mark: 100

<u>Question 1</u> (30 marks)

(a)If A is a square matrix, state the definition of singular, symmetric, the relation between |A|, eigenvalues and the relation between trace A, eigenvalues. 5

(b)If $A=\begin{bmatrix}1 & 2 & 4\\ 1 & 0 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & 1\\ 3 & -3 & 0\end{bmatrix}$, $C=\begin{bmatrix}2 & 0\\ 3 & 1\end{bmatrix}$ 5

Find, if possible A + B , A + C, A.B , C.A , |A.A`|, |A| , |C|.

(c)Find the inverse of $A=\begin{bmatrix}3 & 1 & -1\\ 2 & 2 & -1\\ 2 & 2 & 0\end{bmatrix}$ from its Hamilton equation. 5

(d)Find the eigenvalues and the eigenvectors of the matrix : $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ 8

Also, find $f(A)=\sqrt{A^2+3I}$, $g(A)=A^n$.

(e)Solve the L.S : (i) $x-2y+z=0,\quad 2x+y-z=7,\quad x+2y-2z=5$ 3

(ii) $x-2y+2z=2,\quad -z+y-2x=3,\quad -y-x+z=5$ 4

<u>Question 2</u> (20 marks)

(a)Use the definition to differentiate the function $f(x)=\sqrt{x+1}$. 4

(b)Find the nth derivative for $y=\ln(x^2-5x+6)$. 5

(c)Find the values of the constants **a** and **b** for which the following function is 5

continuous everywhere, $f(x)=\begin{cases} a, & x\le -3\\ \dfrac{9-x^2}{4-\sqrt{x^2+7}}, & |x|<3\\ b, & x\ge 3\end{cases}$

(d)Find y' : (i) $y=(\tan x)^{\sin x}$ (ii) $y=\dfrac{\sqrt{x+2}.(x^2+3).\sin 2x}{\sqrt[3]{2x-5}.(3x+4)^4.\sin x}$ 6

(iii) $x^2+y^2=2xy$

<u>Question 3</u> (30 marks)

(a)Sketch the curve of the function: $f(x)=x^3-3x$. Also, find the value of c that satisfies the mean value theorem in the interval [–1, 1]. 6

(b)Find the limits: (i) $\lim\limits_{x\to 0}\dfrac{3^{\sin x}-1}{x}$ (ii) $\lim\limits_{x\to\infty}(x-\ln x)$ (iii) $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ 6

(c)Find the integrals: 18

(i) $\int (x^3-3^x)\,dx$ (ii) $\int (3+x^3)^2\,dx$ (iii) $\int \dfrac{x+3}{\sqrt{9x-x^2}}\,dx$

(iv) $\int \sin^4 x.\cos^5 x\,dx$ (v) $\int \dfrac{\cos x}{\sin x.\ \sqrt{\ln \sin x}}\,dx$ (vi) $\int \dfrac{\tan x.\ \sec^2 x}{9+\sec^4 x}\,dx$

أنظر السؤال الرابع فى الخلف

Good Luck Dr. Mohamed Eid Dr. Ibrahim Saker Dr. Khaled Elnagar Dr. Reda Abdelkader

Model Answer

Answer of Question 1

(a)(i)The sum of all eigenvalues of a unit matrix of order n equals to n.

(ii)A square matrix A is called non singular if $|A| \neq 0$ if and it is called symmetric if $A = A`$

(iii)If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to $A + \sqrt{A}$.

(iv)A linear system AX = B is called inconsistent if it has no solution.

(v)A linear system AX = B has one solution if rank A = rank G = number of unknowns.

--*5 Marks*

(b)If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

A + C, A.B, C.A and |A| are nor exist.

$A + B = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 0 & 0 \end{bmatrix}$, $A.C = \begin{bmatrix} 4 & -4 & 10 \\ 7 & -2 & 6 \end{bmatrix}$, $|C| = 1 + 8 - 6 = 3$

--*8 Marks*

(c) $\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2 = \lambda^2 - 5\lambda + 4 = 0$. Then $\lambda_1 = 1$, $\lambda_2 = 4$

From the equation, $\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 1$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + 2y = 0, x + 2y = 0

Put x = 2, then the corresponding

eigenvector is : $X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

For : $\lambda_2 = 4$, $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –2x + 2y = 0, x – y = 0

Put y = 1, we get x = 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then $T = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

The diagonal form : $T^{-1} A\, T = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$. Then $T^{-1} \ln A\ T = \begin{bmatrix} \ln 1 & 0 \\ 0 & \ln 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}$

Then $\ln A = T \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix} T^{-1} = \frac{1}{3}\begin{bmatrix} 1.4 & 2.8 \\ 1.4 & 2.8 \end{bmatrix}$

--*8 Marks*

(d)By Calculator, the solution is : (1, 2, 4).

(ii) $G = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ -1 & 1 & -1 & 3 \\ 3 & -3 & 4 & 3 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -2 & -3 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 7 \end{array}\right]$

Rank A = 2, Rank G = 3. No solution.

---*4 Marks*

Dr. Mohamed Eid

Answer of Question 4

a) Since the hypotenuse of the right triangle is constant, therefore $x^2 + y^2 = c^2 \Rightarrow$ $2x + 2yy` = 0 \Rightarrow y` = -x/y$

Area: $A = \frac{1}{2}xy \Rightarrow \frac{dA}{dx} = \frac{1}{2}(xy`+y) = \frac{1}{2}(x(\frac{-x}{y}) + y) = 0 \Rightarrow x^2 = y^2 \Rightarrow x = y$

Therefore the area is maximum when the triangle is isosceles.

b) $y` = \dfrac{[\ell n\, x]^x [\ell n(\ell n x) + x(\frac{1/x}{\ln x})][2^{\ell n(x)-1}] - [\ell n\, x]^x [\frac{1}{x}][2^{\ell n(x)-1}]\, \ell n\, 2}{[2^{\ell n(x)-1}]^2}$

at x = e, slope of the tangent = $y`(1) = 1 - \frac{\ell n\, 2}{e} = \frac{e - \ell n\, 2}{e}$

Therfore the point of contact is (e, 1) and hence the equation of tangent is $\frac{y-1}{x-e} = \frac{e - \ell n\, 2}{e}$

Also equation of normal is $\frac{y-1}{x-e} = \frac{e}{\ell n\, 2 - e}$

c-i) $\lim\limits_{x\to\infty} \frac{5^x + 2^x}{9^x} = \lim\limits_{x\to\infty} \frac{5^x(1 + (\frac{2}{5})^x)}{9^x} = \lim\limits_{x\to\infty} (\frac{5}{9})^x = 0$

ii) This limit of the indeterminate form ∞^0. Let $y = x^{1/\sqrt{x}} \Rightarrow \ell n\, y = \frac{\ell n\, x}{\sqrt{x}}$

Hence L'Hopital rule is ready to be used such that

$\lim\limits_{x\to\infty} \ell n\, y = \lim\limits_{x\to\infty} \frac{\ell n\, x}{\sqrt{x}} = \frac{\infty}{\infty} = \lim\limits_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} = \lim\limits_{x\to\infty} \frac{2}{\sqrt{x}} = 0$.

Therefore $\lim\limits_{x\to\infty} x^{1/\sqrt{x}} = \lim\limits_{x\to\infty} e^{\ell n\, y} = e^{\lim\limits_{x\to\infty} \ell n\, y} = e^0 = 1$

iii) $\lim\limits_{x\to 0} [\frac{1}{x} - \frac{1}{e^x - 1}] = \lim\limits_{x\to 0} [\frac{e^x - 1 - x}{x(e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x - 1}{xe^x + (e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x}{xe^x + 2e^x}] = \frac{1}{2}$

iv) $\lim\limits_{x\to\infty} e^{-x}\, \ell n\, x = \lim\limits_{x\to\infty} \frac{\ell n\, x}{e^x} = \lim\limits_{x\to\infty} \frac{1}{xe^x} = 0$

Dr. Khaled EI Nagar